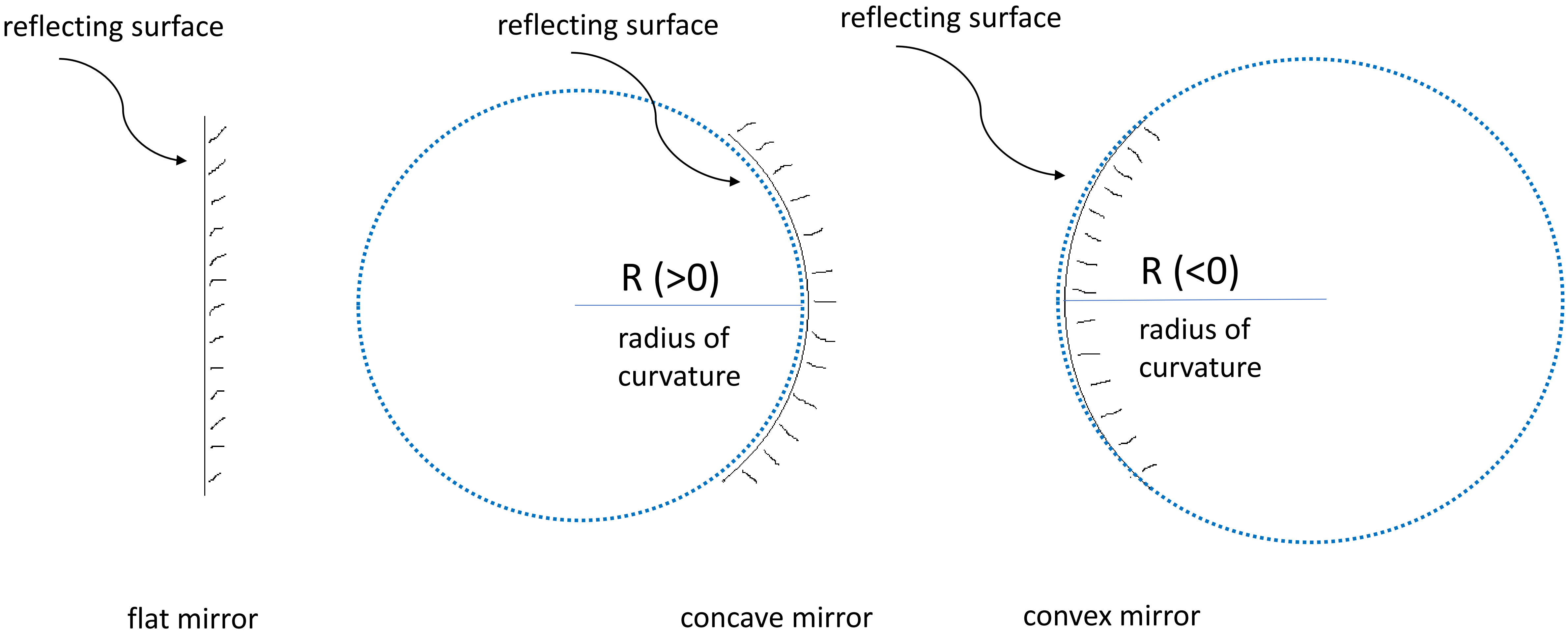
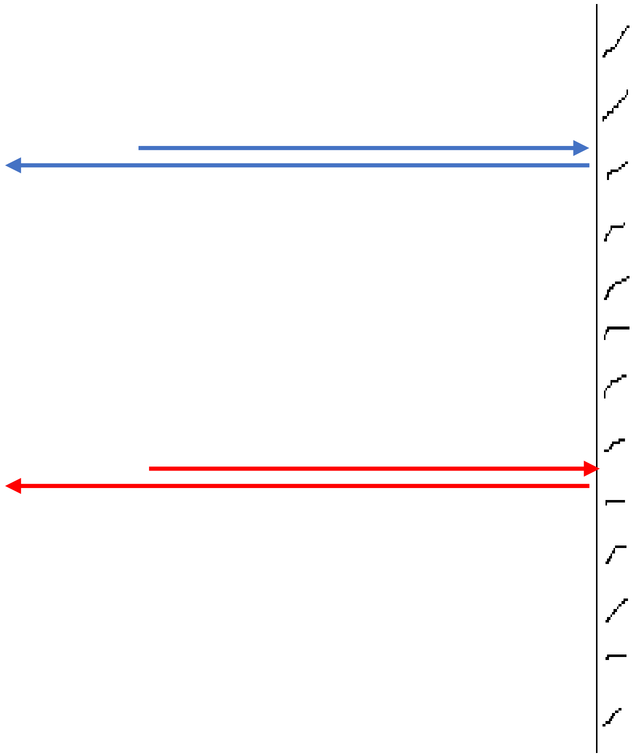
E.2 Mirrors

There are three kinds of mirrors out there in life:

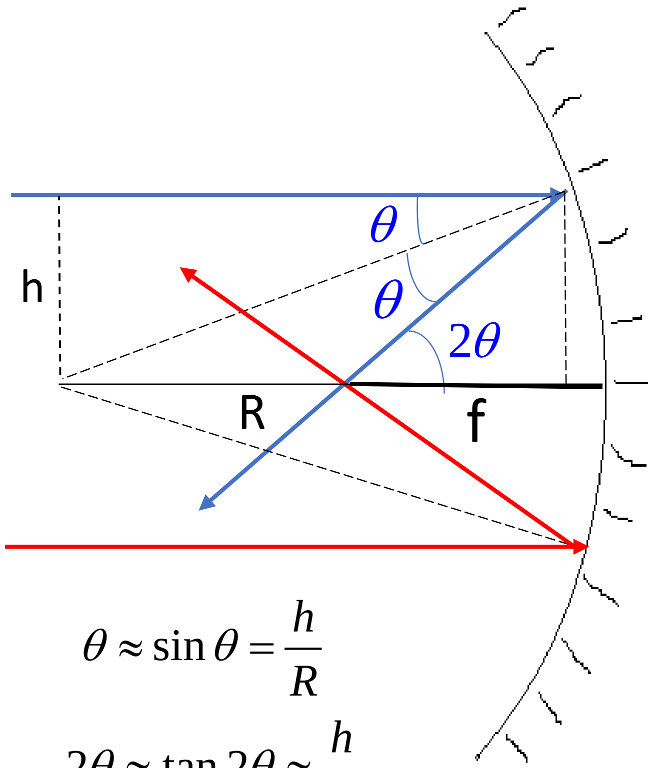


E.2 Mirrors

One of the main characteristics of a mirror is its focal point, f, i.e. the point at which parallel light rays converge, after hitting the mirror. So the first order of business is to discover where this is….we'll liberally use the small angle approximation: $\Theta \approx \sin\Theta \approx \tan\Theta$ to help.



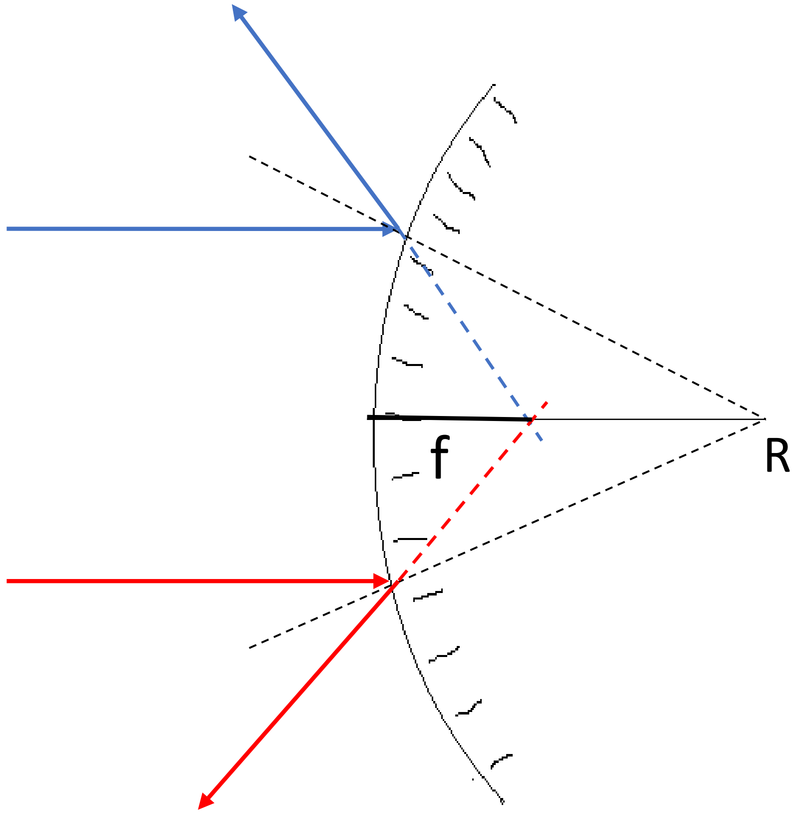
$$f = \infty$$



$$\theta \approx \sin\theta = \frac{h}{R}$$

$$2\theta \approx \tan 2\theta \approx \frac{h}{f}$$

$$2\left(\frac{h}{R}\right) \approx \frac{h}{f} \longrightarrow f \approx \frac{R}{2}$$

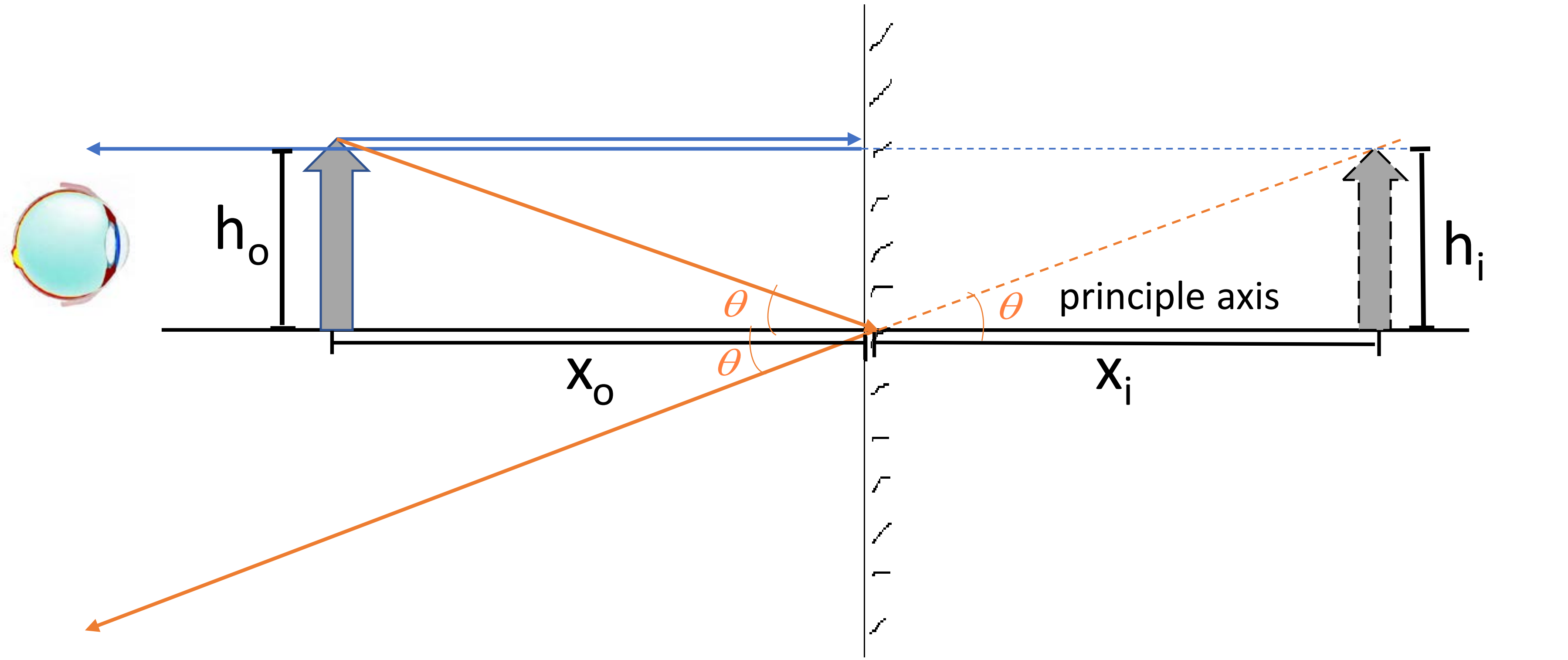


$$f \approx \frac{R}{2}$$

E.2 Mirrors

region where light came from: $x_o > 0$

region where light went: $x_i > 0$



It's clear that $h_i = h_o$. So,

$$m \equiv \frac{h_i}{h_o} = 1$$

And since the angles are equal, and the heights are equal, the distances are equal:

$$x_i = -x_o$$

So note that the image is in the negative image distance region, so x_i is *negative*.

E.2 Mirrors

region where light came from: $x_o > 0$

region where light went: $x_i > 0$

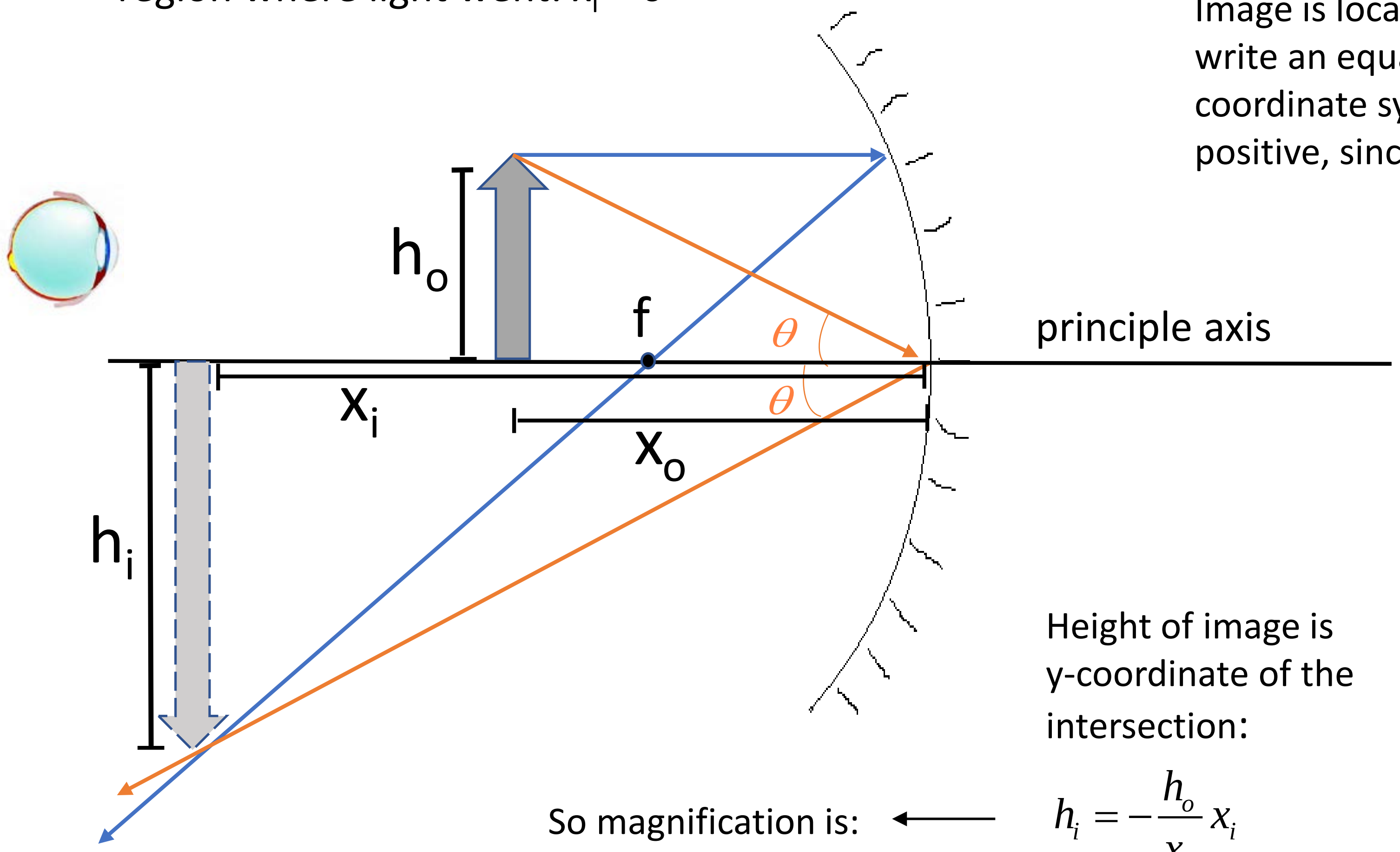


Image is located at the intersection of the two reflected rays. We'll write an equation for them and see where this is. Placing an invisible coordinate system at the center of the mirror, and treating left as positive, since x_i is positive in that direction…

Blue reflection:

$$y_{blue} = y_0 + slope \cdot x = h_0 - \frac{h_o}{f}x$$

Orange reflection:

$$y_{orange} = y_0 + slope \cdot x = 0 - \frac{h_o}{x_o}x$$

Intersection:

$$y_{blue} = y_{orange} \longrightarrow \cancel{h_0} - \frac{\cancel{h_o}}{f}x_i = -\frac{\cancel{h_o}}{x_o}x_i$$

$$1 - \frac{1}{f}x_i = -\frac{1}{x_o}x_i$$

$$\boxed{\frac{1}{x_o} + \frac{1}{x_i} = \frac{1}{f}} \longleftarrow 1 - \frac{1}{f}x_i = -\frac{1}{x_o}x_i$$

Height of image is y-coordinate of the intersection:

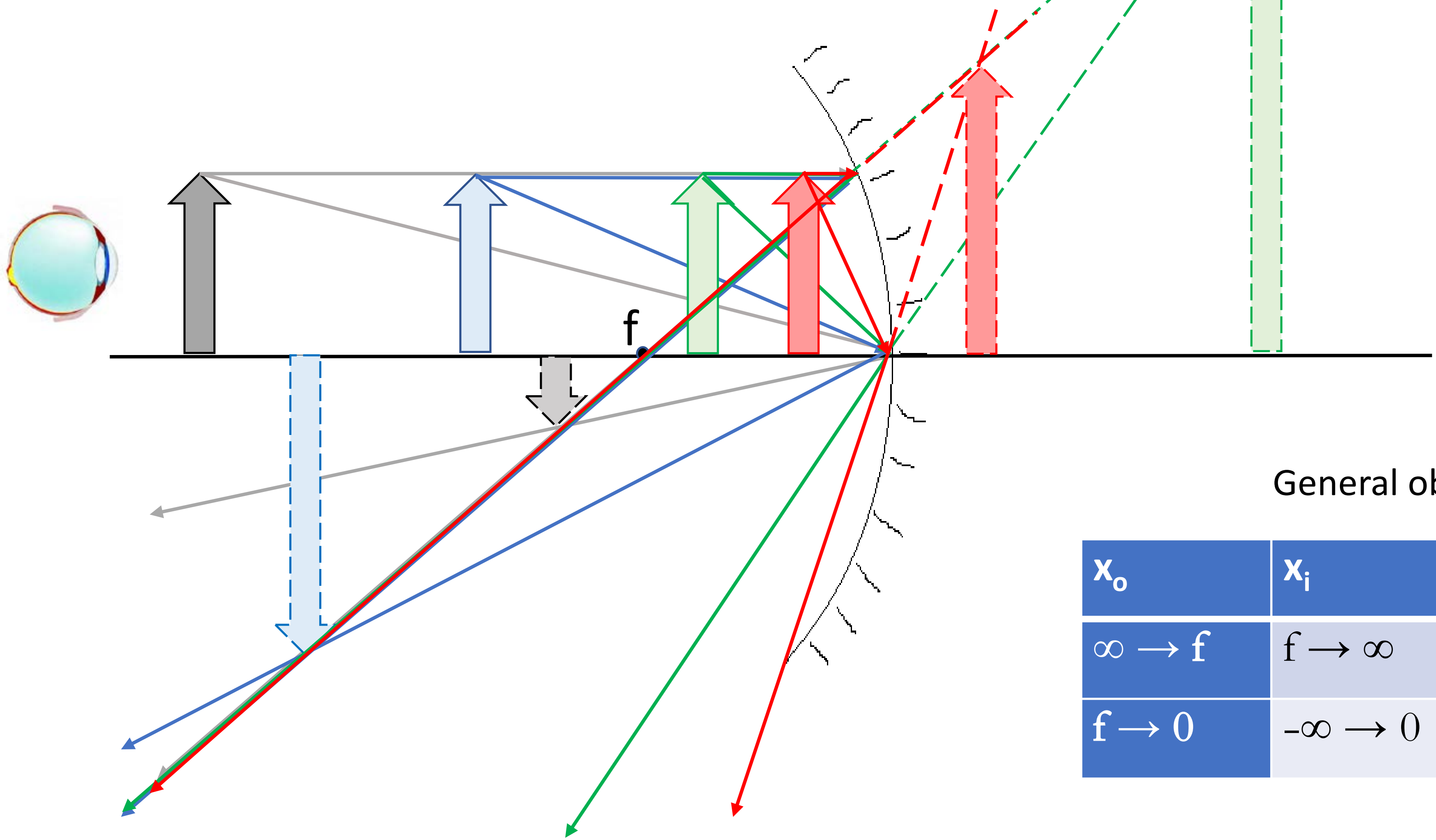
$$h_i = -\frac{h_o}{x_o}x_i$$

So magnification is: $\longleftarrow$

$$\boxed{m \equiv \frac{h_i}{h_o} = -\frac{x_i}{x_o}}$$

E.2 Mirrors

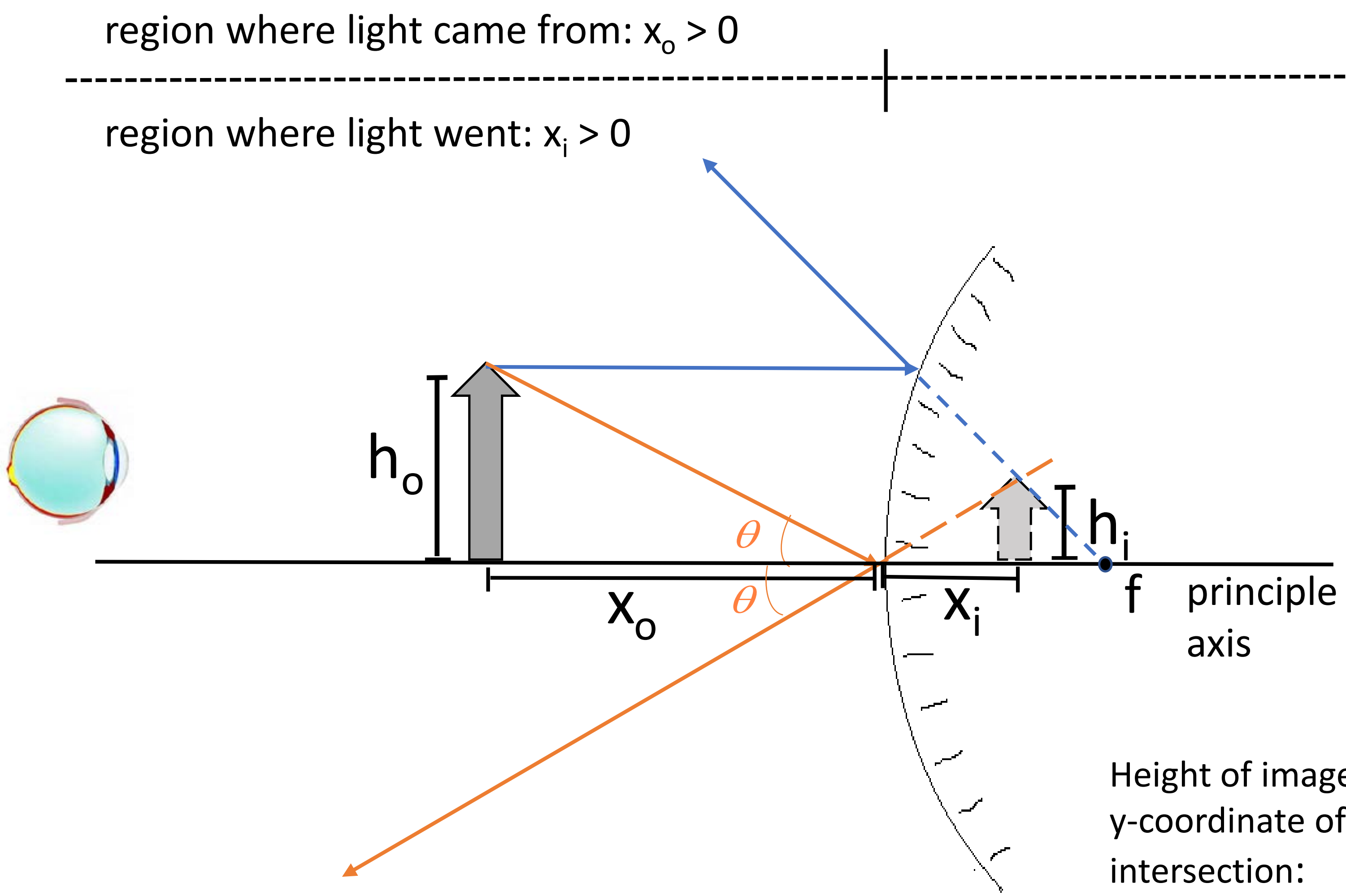
Let’s step back and survey the kinds of images we can get from a concave mirror:



General observations:

x_o	x_i	m	type
$\infty \to f$	$f \to \infty$	$0 \to -\infty$	real
$f \to 0$	$-\infty \to 0$	$\infty \to 1$	virtual

E.2 Mirrors



Locating image via intersection of the two (pseudo) reflected rays again, treating left as positive and noting f is negative…

Blue reflection:

$$y_{blue} = y_0 + slope \cdot x = h_0 + \frac{h_o}{-f}x$$

Orange reflection:

$$y_{orange} = y_0 + slope \cdot x = 0 - \frac{h_o}{x_o}x$$

Intersection:

$$y_{blue} = y_{orange} \longrightarrow \cancel{h_0} + \frac{\cancel{h_o}}{-f}x_i = -\frac{\cancel{h_o}}{x_o}x_i$$

$$\downarrow$$

$$1 + \frac{1}{-f}x_i = -\frac{1}{x_o}x_i \longrightarrow \boxed{\frac{1}{x_o} + \frac{1}{x_i} = \frac{1}{f}}$$

Height of image is y-coordinate of the intersection:

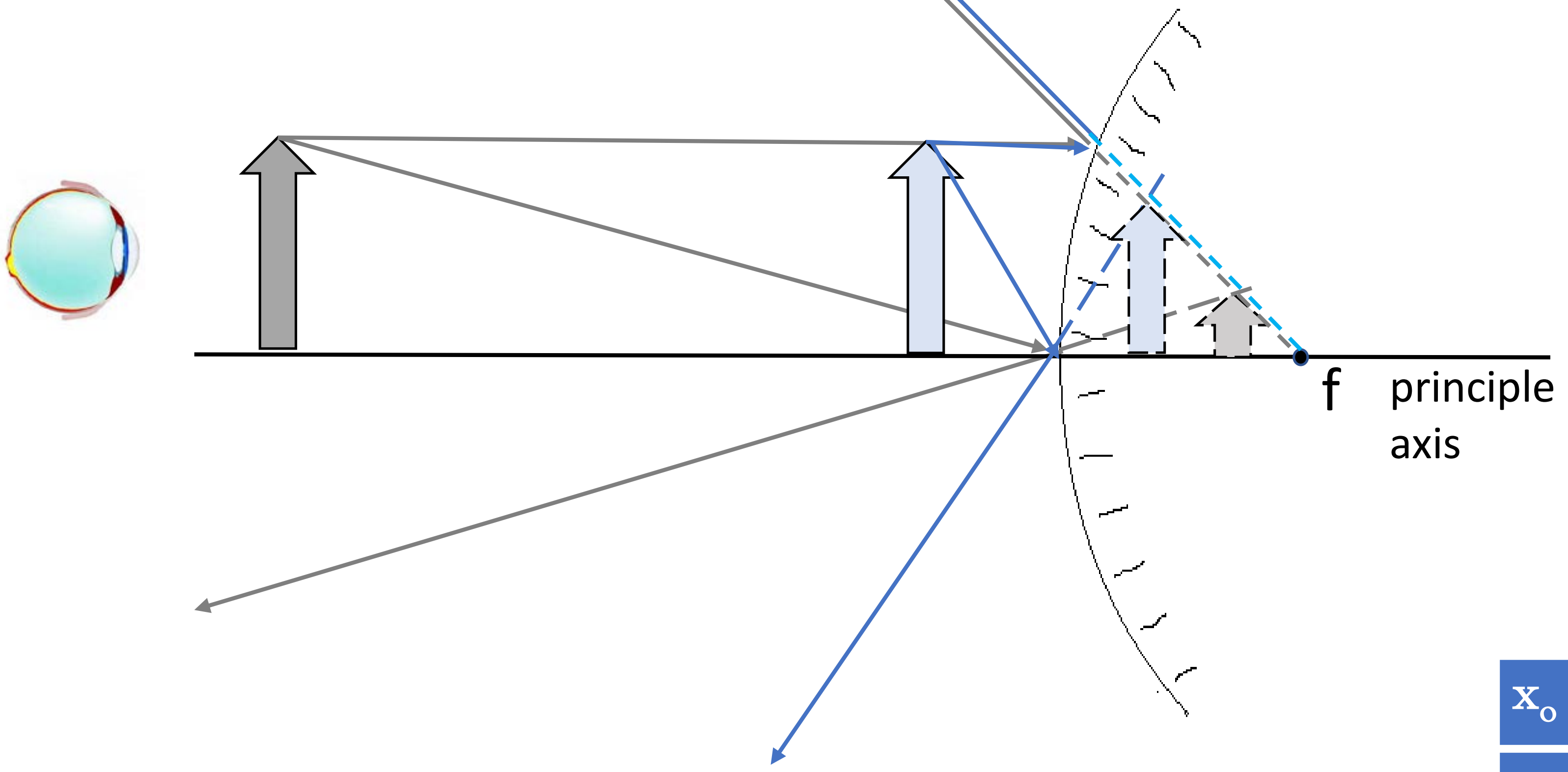
$$h_i = -\frac{h_o}{x_o}x_i$$

So magnification is: $\boxed{m \equiv \frac{h_i}{h_o} = -\frac{x_i}{x_o}}$

So same equations as for convex lens ☺

E.2 Mirrors

Let's step back and survey the kinds of images we can get from a concave mirror:



General observations:

x_o	x_i	m	type
$\infty \rightarrow 0$	$-f \rightarrow 0$	$0 \rightarrow 1$	virtual

E.2 Mirrors

Standing 20cm from an ATM mirror, you see yourself magnified by factor of 1/10.
What is the focal length of the mirror? What's its radius of curvature? How far behind the mirror is your image?

$$\frac{1}{x_o}+\frac{1}{x_i}=\frac{1}{f}$$

$$m=1/10$$

$$-\frac{x_i}{x_o}=1/20$$

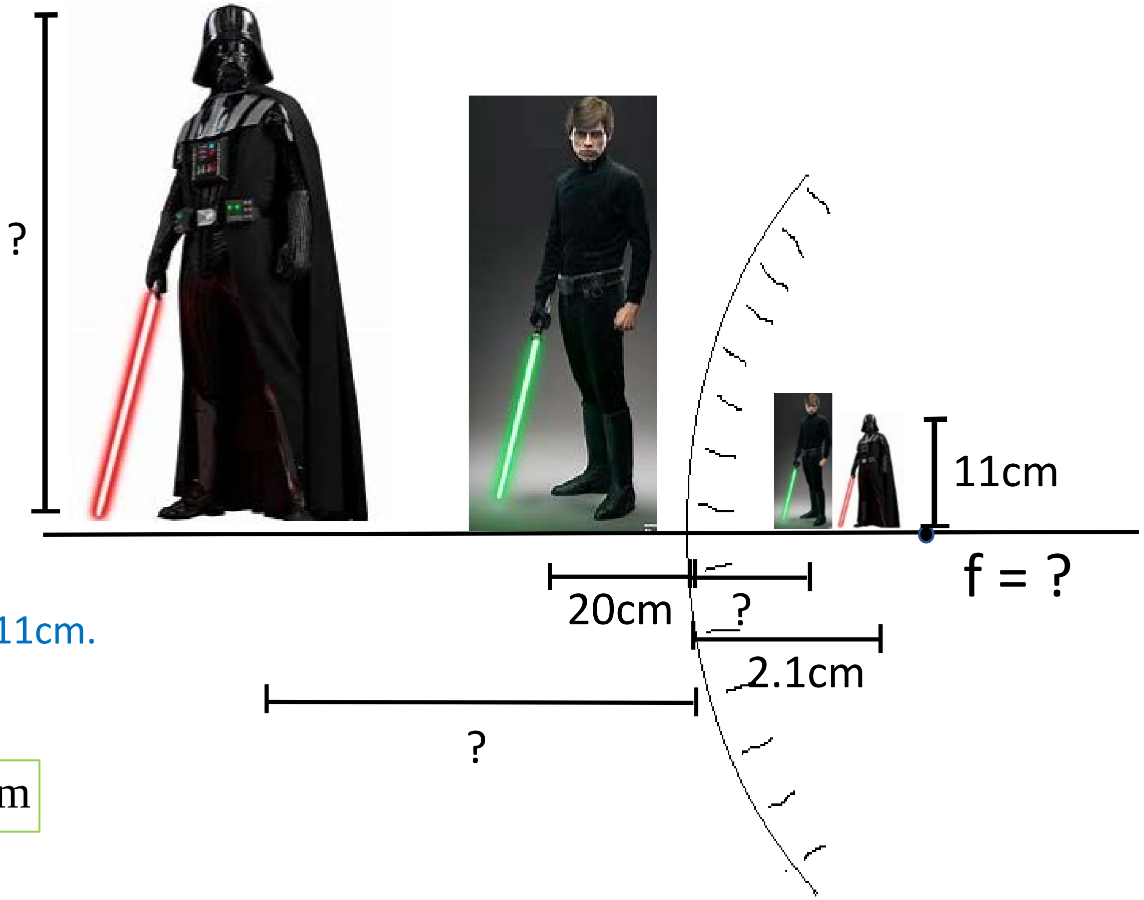
$$\frac{1}{20\text{cm}}+\frac{1}{x_i}=\frac{1}{f}$$

$$-\frac{x_i}{20\text{cm}}=1/10$$

$$\boxed{x_i=-(20\text{cm})(1/10)=-2\text{cm}}$$

$$\frac{1}{20\text{cm}}+\frac{1}{-2\text{cm}}=\frac{1}{f}$$

$$\boxed{f=-2.2\,\text{cm}} \longrightarrow \boxed{R=2f=-4.4\text{cm}}$$



Darth Vader's image shows up 2.1cm behind the mirror with height 11cm.
Where is he, and what is his height?

$$\frac{1}{x_o}+\frac{1}{x_i}=\frac{1}{f} \longrightarrow \frac{1}{x_o}+\frac{1}{-2.1\text{cm}}=\frac{1}{-2.2\text{cm}} \longrightarrow \boxed{x_o=46\text{cm}}$$

$$m=\frac{h_i}{h_o} \longrightarrow -\frac{x_i}{x_o}=\frac{h_i}{h_o} \longrightarrow -\frac{-2.1\text{cm}}{46\text{cm}}=\frac{11\text{cm}}{h_o} \longrightarrow \boxed{h_o=241\text{cm}=2.41\text{m}}$$